Channel Coding Techniques for Communication over Networks and Channels with Memory

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University of California San Diego

Ph.D. Defense

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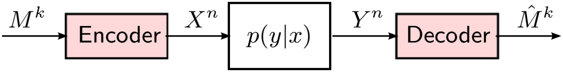
Point-to-Point Communication

- Shannon's model



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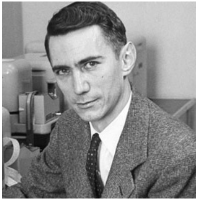

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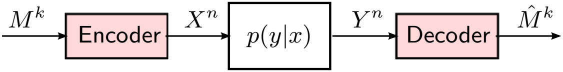

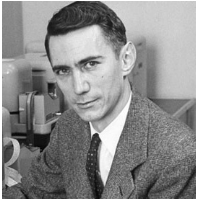

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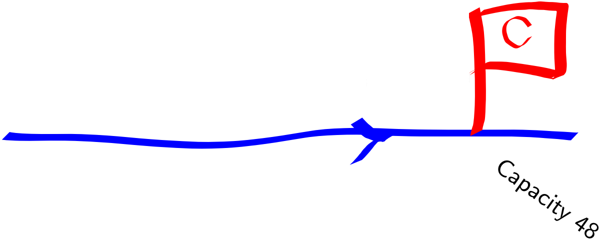
P2P Channel Coding Theorem [Shannon 1948]

Channel $p(y\,|\,x)$ with **capacity** C:

- A family of codes with vanishing P_e exists only if $R \leq C$.
- For any $R < C$, a family of codes with vanishing P_e exists.

Road to Capacity



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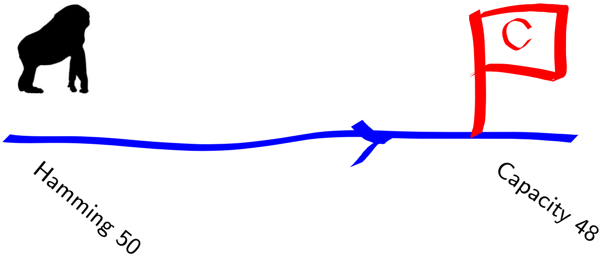

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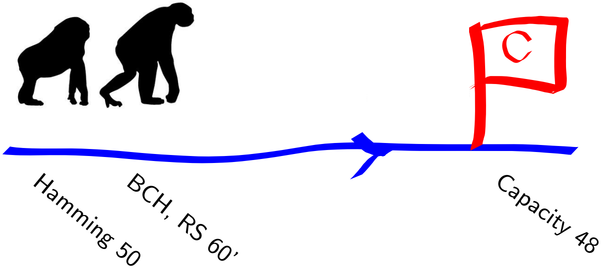

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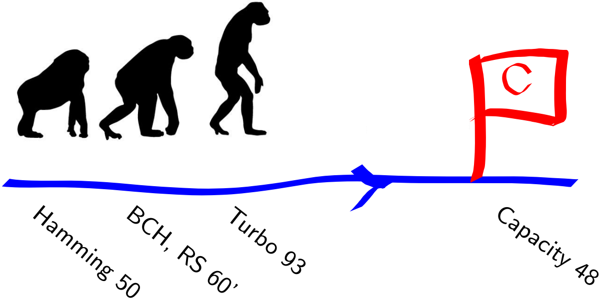

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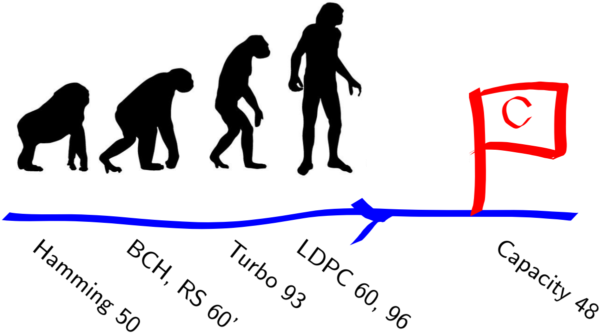

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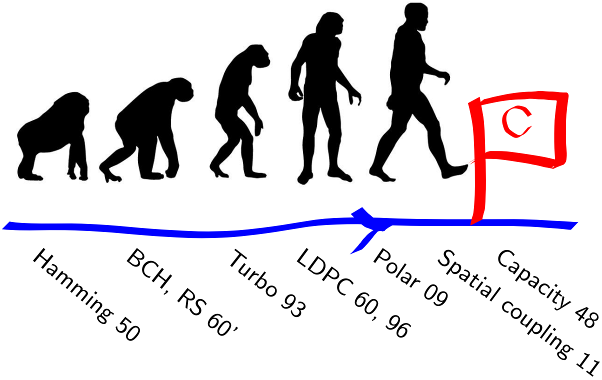

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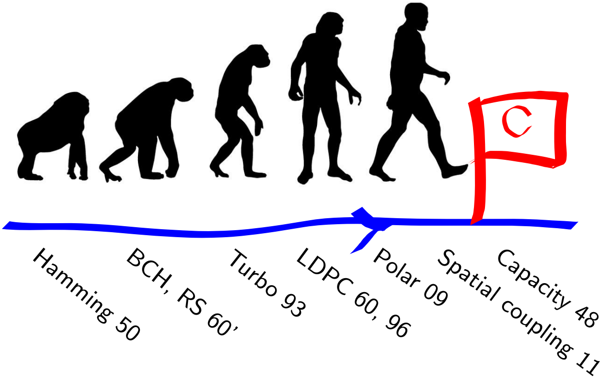

Approach capacity for all binary point-to-point memoryless symmetric (BMS) channels!

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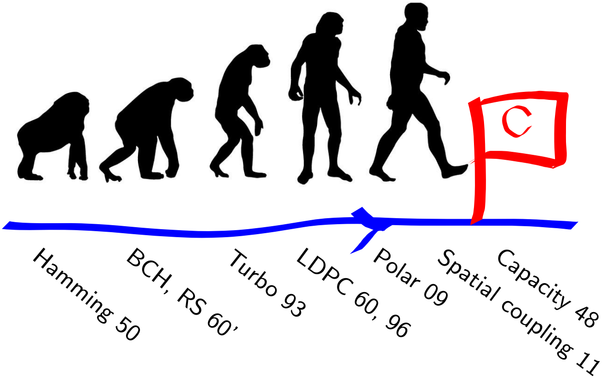

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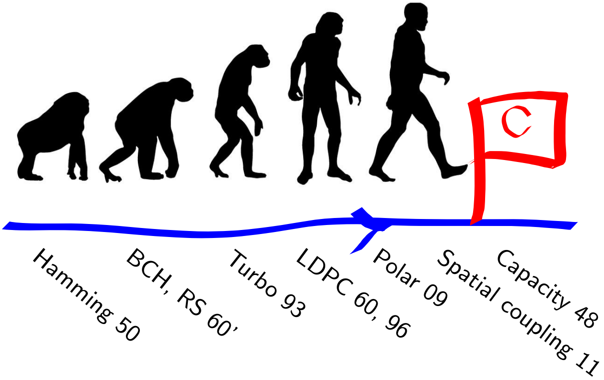

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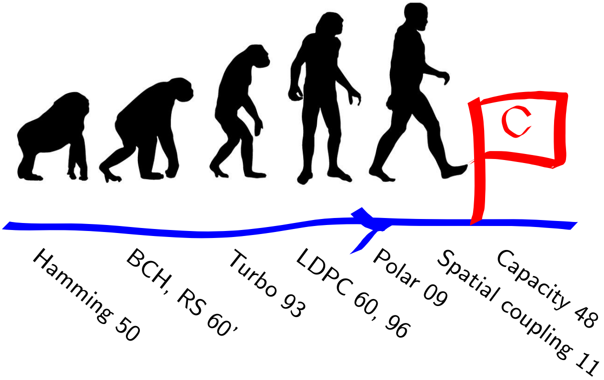

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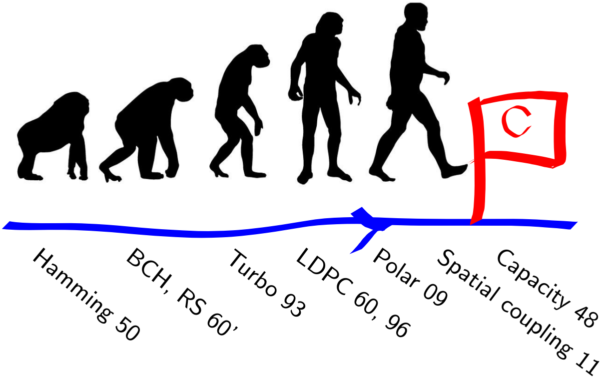

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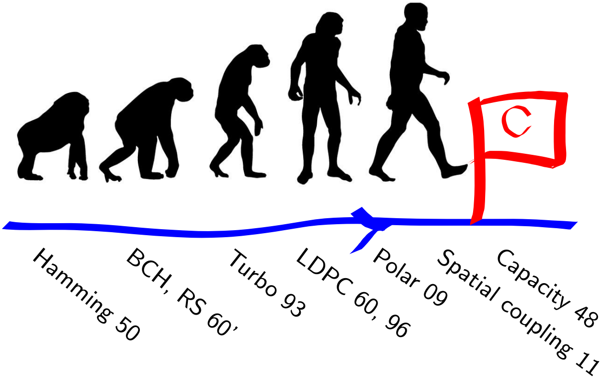

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This Dissertation: Networks and Channels with Memory

I. Coding over networks: A Lego-brick approach

II. Joint channel estimation and polar coding over channels with memory

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- Marton coding over broadcast channels
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- Pilot arrangement pattern that uses code structure
- Finite-state Markov channels
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Coding over Networks: A Lego-Brick Approach

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Network information theory: Characterizes achievable rates for network communication.

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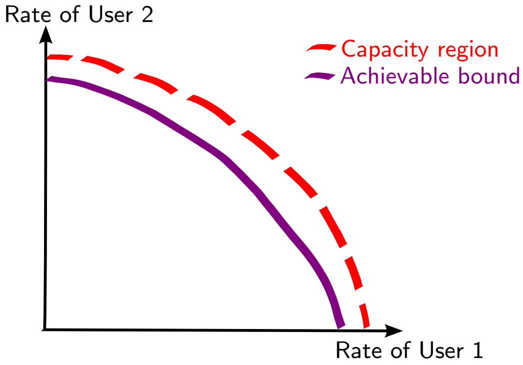

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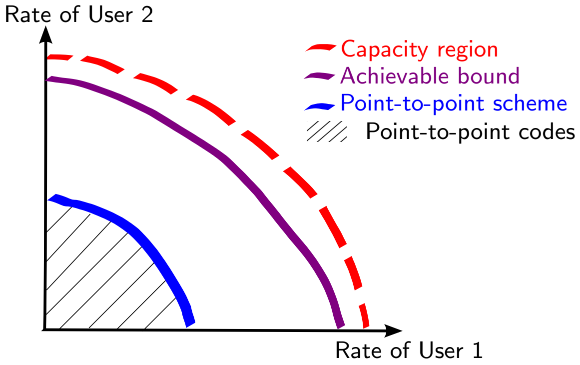

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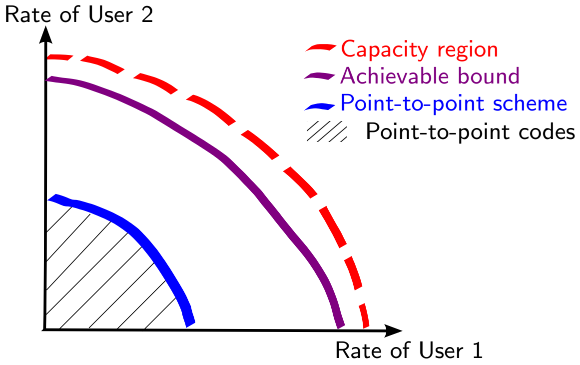

Goal: Construct low-complexity coding schemes over networks!

Previous Work

- Polar codes
  - Slepian–Wolf coding [Arıkan 2012]
  - Lossy source coding of a symmetric source [Korada–Urbanke 2010]
  - Multiple access channels [Şaşoğlu–Telatar–Yeh 2010, Abbe–Telatar 2012]
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- Lattice codes
  - Gaussian channels with Gaussian state (dirty paper coding) [Erez–Shamai–Zamir 2005]

Lego-Brick Approach to Coding

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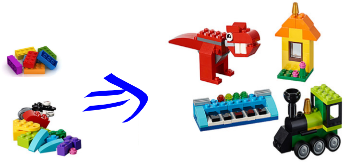
"Lego-brick" approach to coding

Assemble codes in one communication setting $\Longrightarrow$ A code in a different setting



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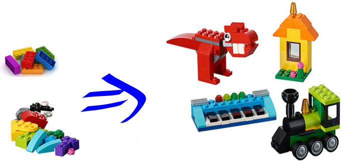

For a given coding problem,

- What "Lego bricks" to assemble, and what properties should they satisfy?
- How to assemble Lego bricks?
- How do performance guarantees translate?

Lego Bricks

- $p(y|x)$ is symmetric if $\exists\, \pi : \mathcal{Y} \to \mathcal{Y}$ s.t. $\pi^{-1} = \pi$ and $p(y|0) = p(\pi(y)|1)$, $\forall y$
  - BSC: $\pi(y) = y \oplus 1$
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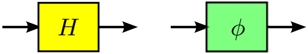

  - P2P code (H, ϕ) for BMS channel $p(y\,|\,x)$
  - Parity-check matrix H, decoder ϕ
  - Dimension k, blocklength n
  - Probability of error ϵ
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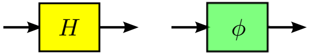

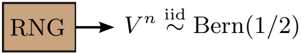

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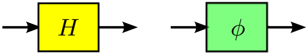

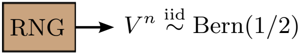

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$$\widetilde{H} \triangleq \begin{bmatrix} \mathbf{0} \\ B^{-1} H \end{bmatrix}.$$

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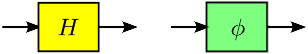

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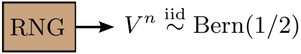

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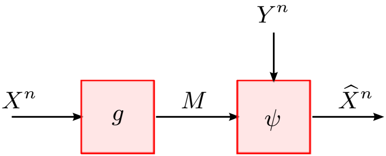
Example: Slepian–Wolf Problem

- Slepian–Wolf problem $p(x, y)$



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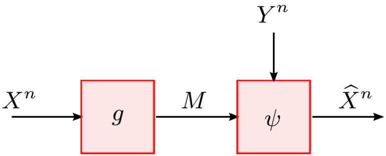

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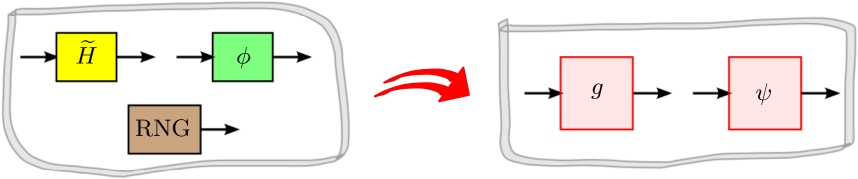

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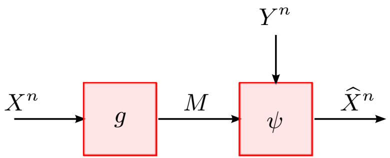

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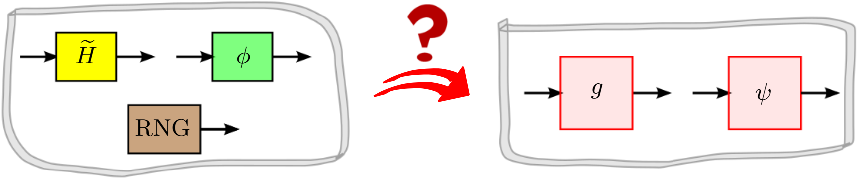

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Example: P2P code $\to$ Slepian–Wolf code [Wang–Kim 2015]

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$$\bar{X}^n \overset{\text{iid}}{\sim} \text{Bern}(1/2) \quad \Longrightarrow \quad \bar{X}^n \oplus \widetilde{H}\bar{X}^n \sim \text{Unif}(\mathcal{C}).$$

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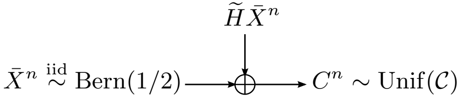

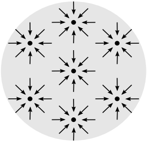

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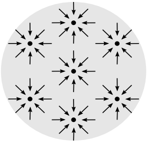

2) **"Symmetrization"**:

Lemma [Chen et al. 2009]

If $V \sim \text{Bern}(1/2) \perp\!\!\!\perp (X,Y)$, then $\bar{p}(y,v\,|\,x) := p_{X,Y}(x \oplus v, y)$ is symmetric

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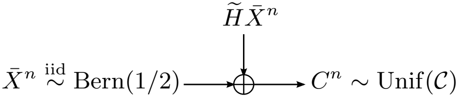
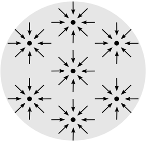
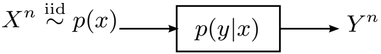




2) **“Symmetrization”**:

Lemma [Chen et al. 2009]

If $V \sim \text{Bern}(1/2) \perp\!\!\!\perp (X,Y)$, then $\bar{p}(y,v\,|\,x) := p_{X,Y}(x \oplus v, y)$ is symmetric



Example: P2P code → Slepian–Wolf code [Wang–Kim 2015]

1) **"Codifying"**:

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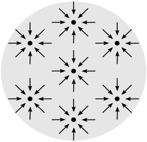

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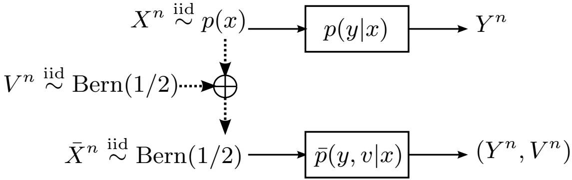

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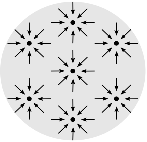

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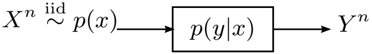

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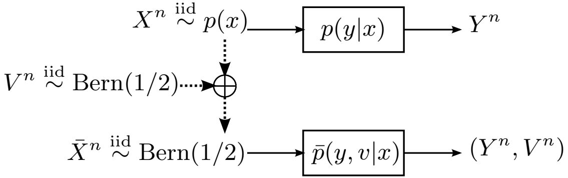

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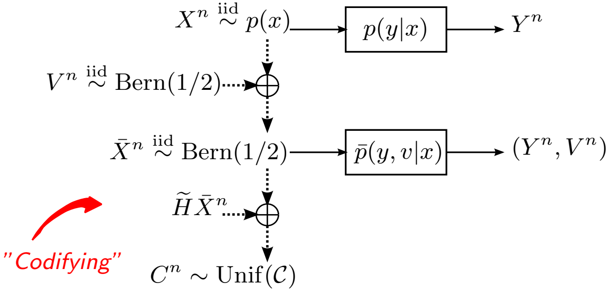

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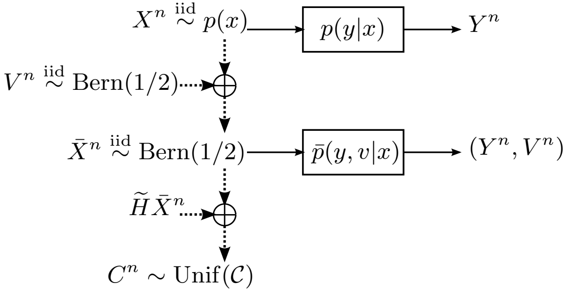

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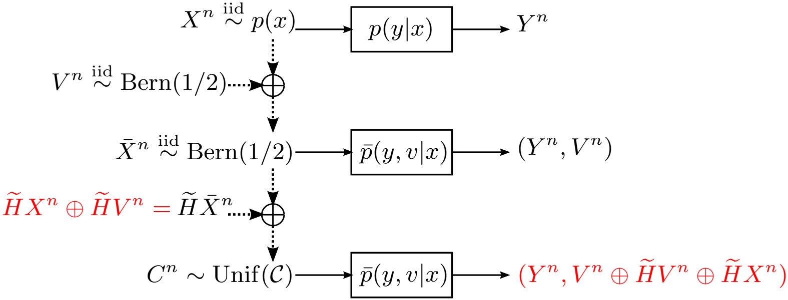

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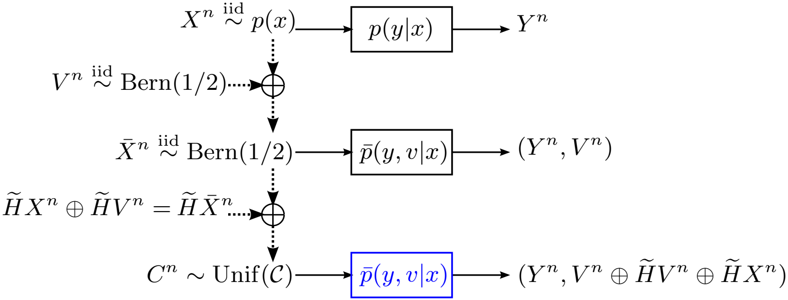

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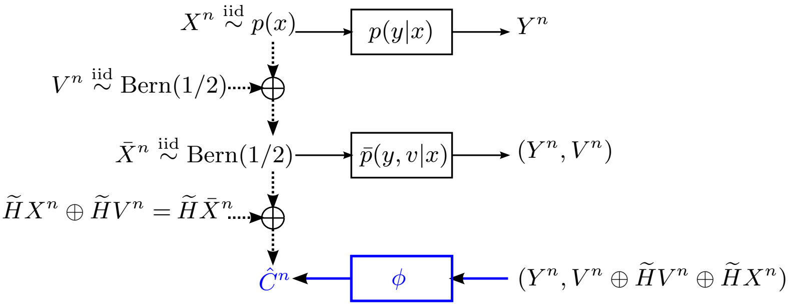

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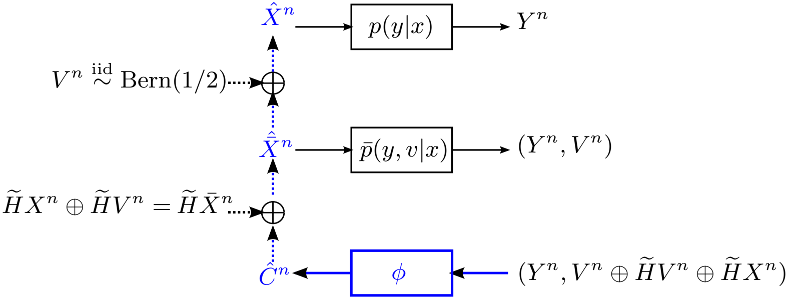

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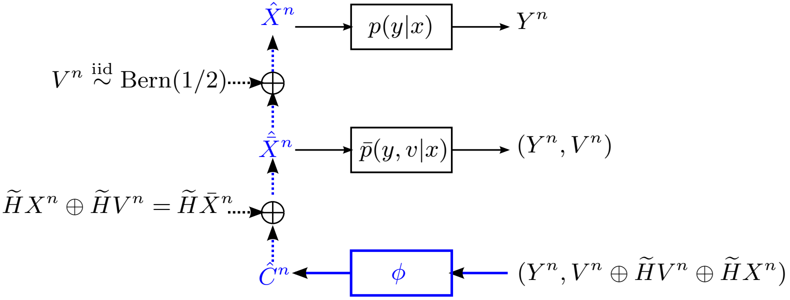

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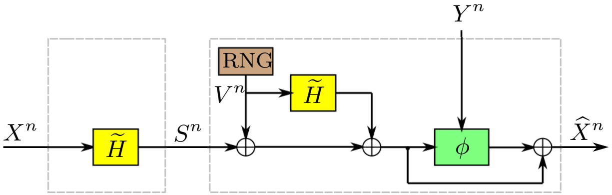

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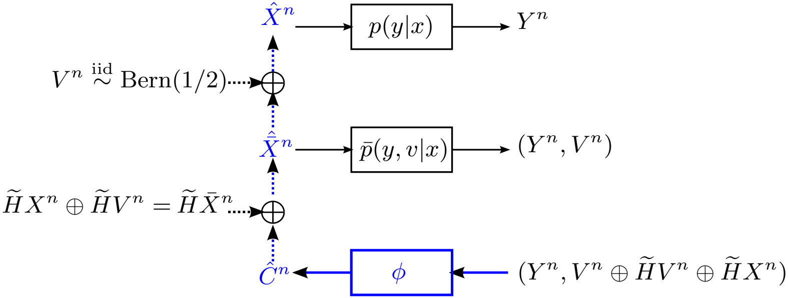

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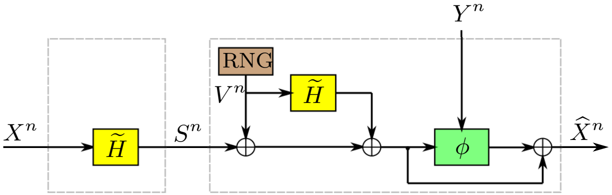

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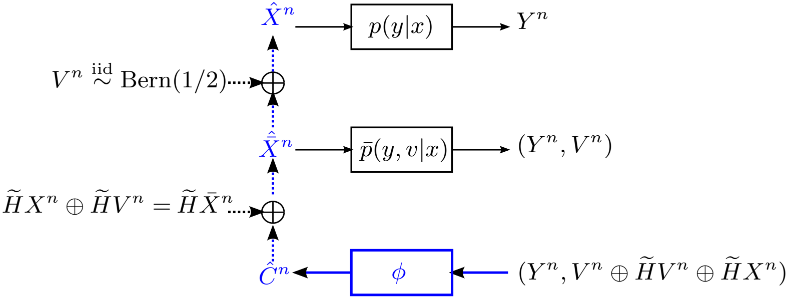

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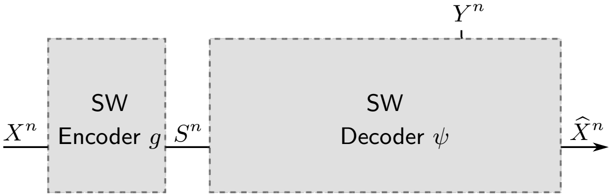

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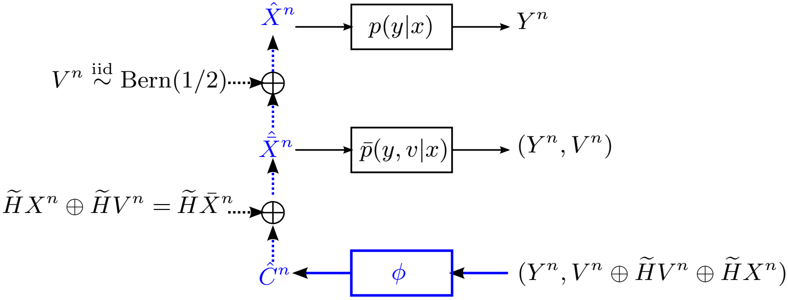

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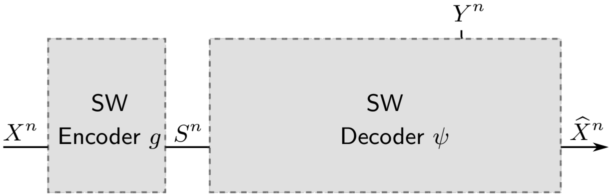

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From Slepian–Wolf to Coding over Networks

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P2P Code for BMS Channel → Slepian–Wolf Code

Properties

Linearity
Error Probability

From Slepian–Wolf to Coding over Networks

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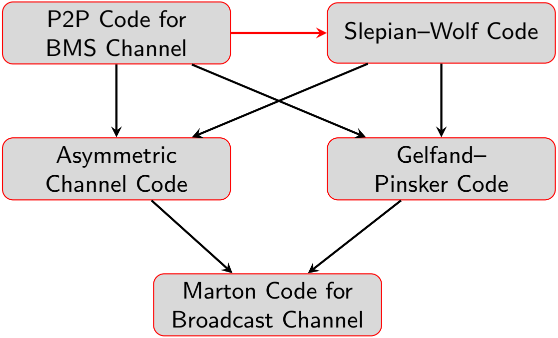

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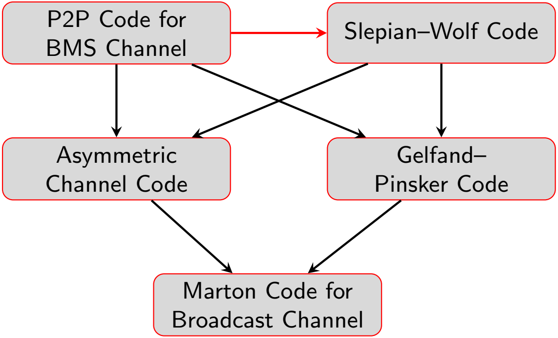

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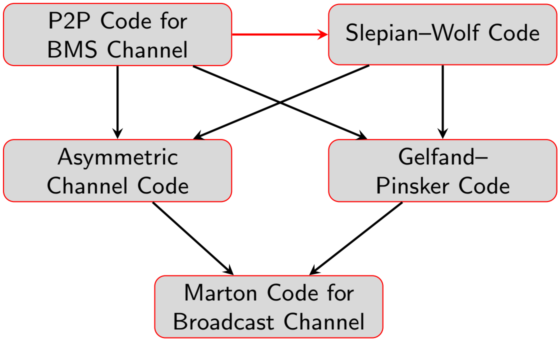

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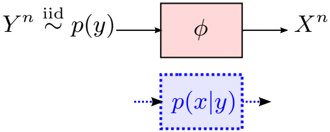

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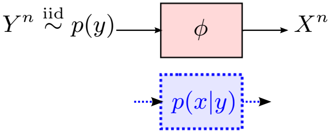

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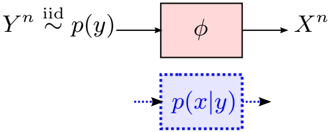

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- Necessary & sufficient condition for vanishing δ: $R > 1 - H(X\,|\,Y)$
  - E.g., random codes [Bennett et. al 2002], polar codes [Korada–Urbanke 2010]

Nested Linear Codes

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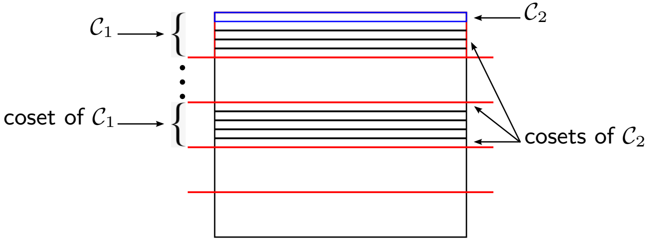

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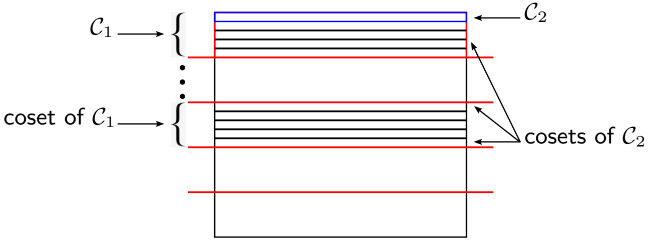

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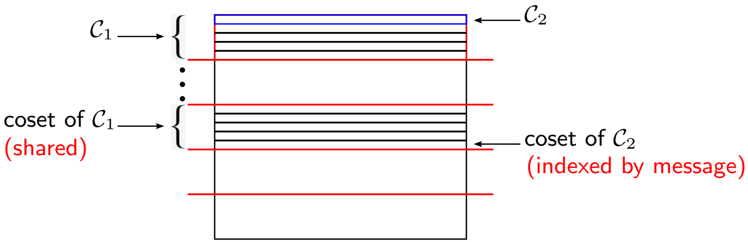

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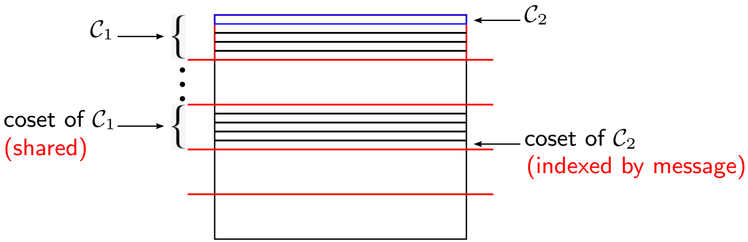

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- Note: We can choose H_1, H_2 s.t. $H_2 = \begin{bmatrix} H_1 \\ Q \end{bmatrix}$ for some matrix Q

P2P Code + Slepian–Wolf Code $\longrightarrow$ Asymmetric Channel Code

- Goal: Code for asymmetric channel $p(y|x)$
- Approach: Target $p(x) \sim \mathrm{Bern}(\alpha)$ for some given α.
- Lego bricks:
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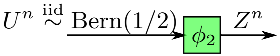

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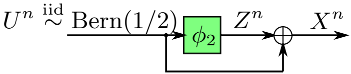

$$X^n = \phi_2(U^n) \oplus U^n$$

$$d_{\mathrm{TV}}\left(q_{U^n,Z^n}, \prod \mathrm{DSBS}(\alpha)\right) = \delta \quad \Longrightarrow \quad d_{\mathrm{TV}}\left(q_{X^n}, \prod \mathrm{Bern}(\alpha)\right) \le \delta$$

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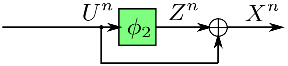

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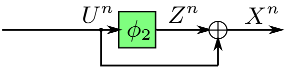
  3. Two codes are nested s.t. $H_2 = \begin{bmatrix} H_1 \\ Q \end{bmatrix}$.

$$\begin{bmatrix} \mathbf{0} \\ V_1^{n-k_1} \\ M^{k_1-k_2} \end{bmatrix}$$



- $V_1^{n-k_1} \overset{\mathrm{iid}}{\sim} \mathrm{Bern}(1/2)$ is shared between encoder and decoder
- $M^{k_1-k_2}$ is the message

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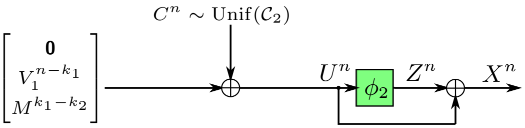

$U^n \overset{\text{iid}}{\sim} \mathrm{Bern}(1/2)$

- $V_1^{n-k_1} \overset{\text{iid}}{\sim} \mathrm{Bern}(1/2)$ is shared between encoder and decoder
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P2P Code + Slepian–Wolf Code $\longrightarrow$ Asymmetric Channel Code

- Goal: Code for asymmetric channel $p(y|x)$
- Approach: Target $p(x) \sim \mathrm{Bern}(\alpha)$ for some given α.
- Lego bricks:
  1. (k_2, n) P2P code (H_2, ϕ_2) for $\mathrm{BSC}(\alpha)$ with decoding distance δ
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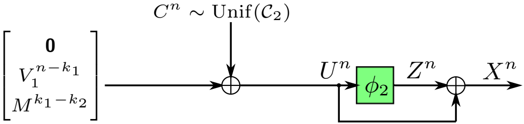

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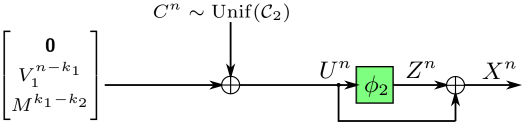

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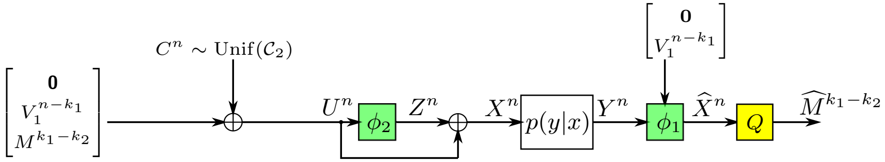

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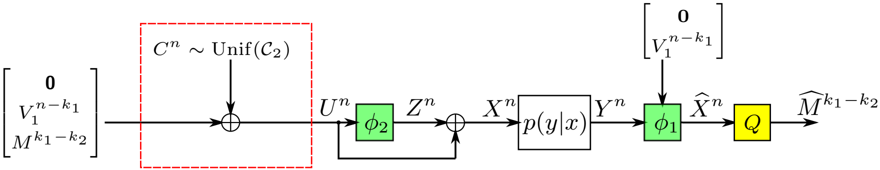

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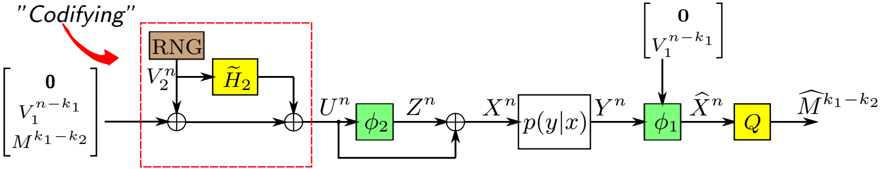

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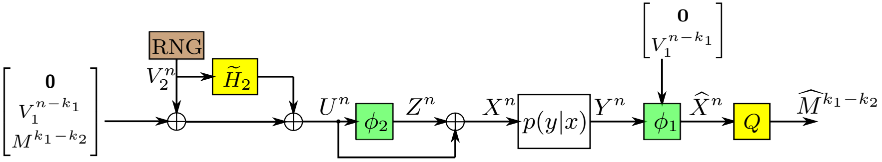

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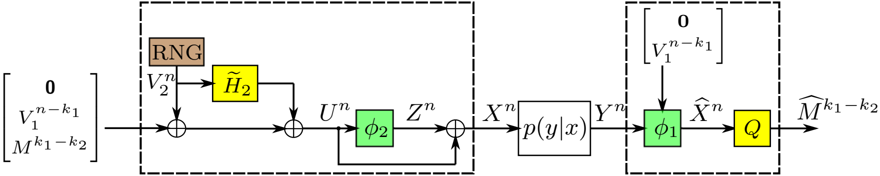

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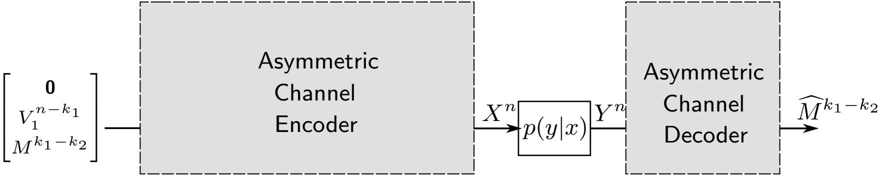

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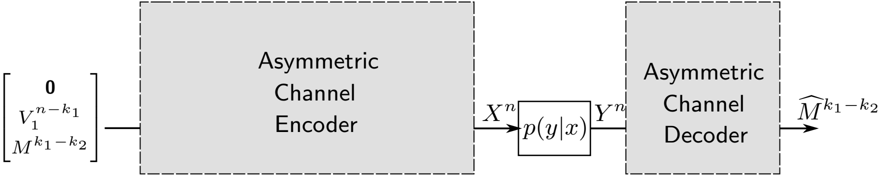

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Theorem

$R = \frac{k_1 - k_2}{n}$

$P_e \le \epsilon + \delta$

Rate Achievability

- $\exists$ a sequence of Slepian–Wolf codes for $p(x, y)$ s.t. $\epsilon \to 0$ if and only if

$$\frac{n - k_1}{n} > H(X \mid Y)$$

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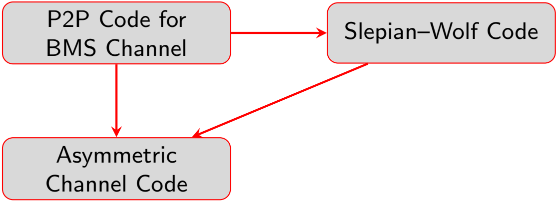
- Rate $R = \frac{k_1 - k_2}{n}$ can be made arbitrarily close to $I(X;Y) = H(X) - H(X\,|\,Y)$.

Outline

- This talk:



Properties

Nested Linearity
Error Probability
Decoding Distance

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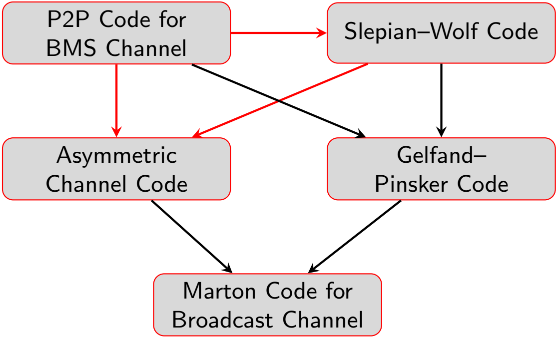

Properties

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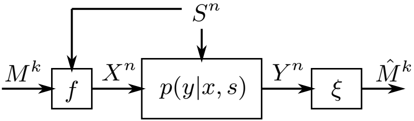
Gelfand–Pinsker Coding

- Channel with state S^n available noncausally only at the encoder



$S^n \overset{\text{iid}}{\sim} p(s)$

Encoder f, Decoder ξ

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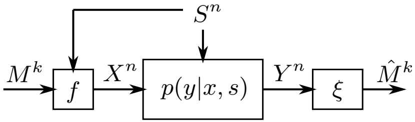

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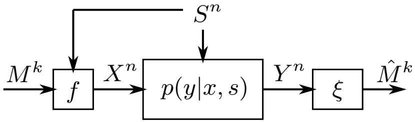

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- Approach: Target a conditional distribution $p(x\,|\,s)$
- This completely defines $p(x, s) = p(s)p(x\,|\,s)$

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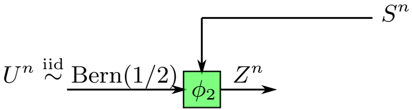

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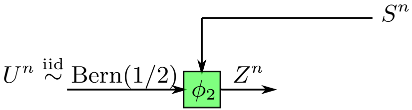

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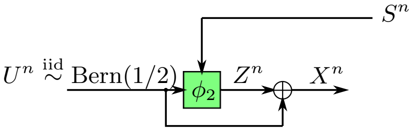

$$X^n = \phi_2(S^n, U^n) \oplus U^n$$

Lemma

$$d_{\text{TV}}\left(q_{X^n,S^n}, \prod p(x, s)\right) \le \delta$$

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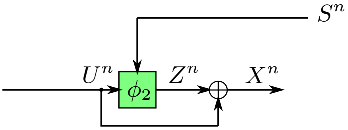

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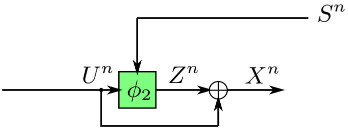

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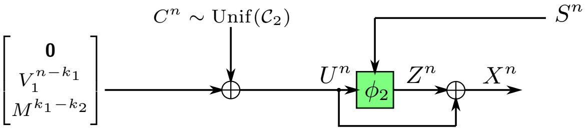

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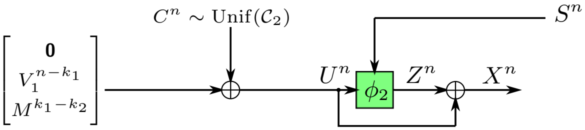

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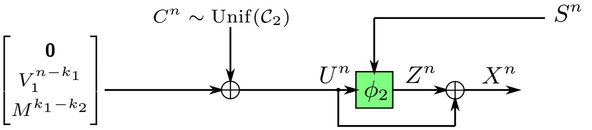

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  1. (k_2, n) P2P code (H_2, ϕ_2) with decoding distance δ for
  $$\bar{p}(s, v \,|\, x) \triangleq p_{X,S}(x \oplus v, s)$$
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  $$p(x, y) = \sum_s p(x, s) p(y \,|\, x, s)$$
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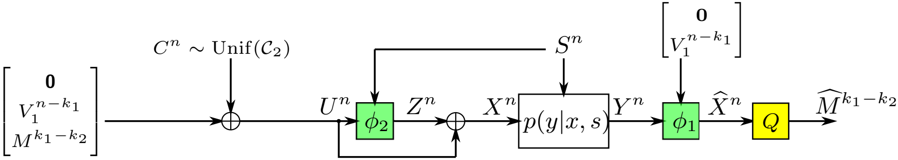
$\bar{p}$ is the *"symmetrized channel"* of $p(x, s)$



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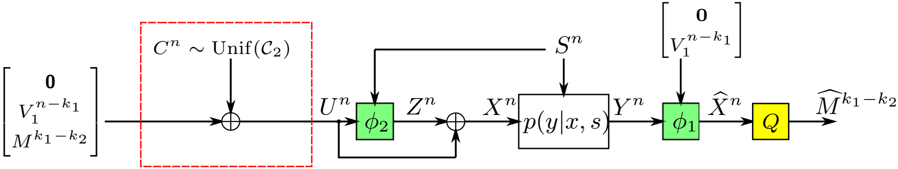

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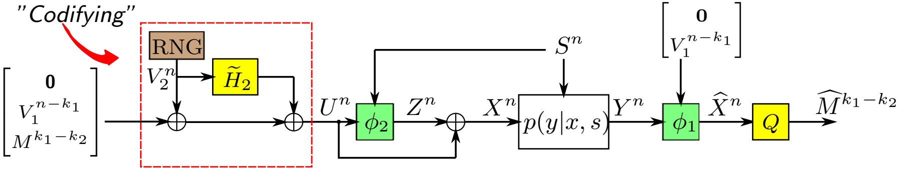

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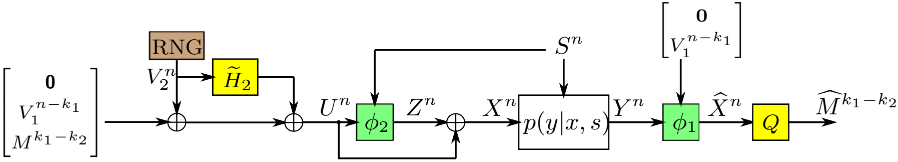

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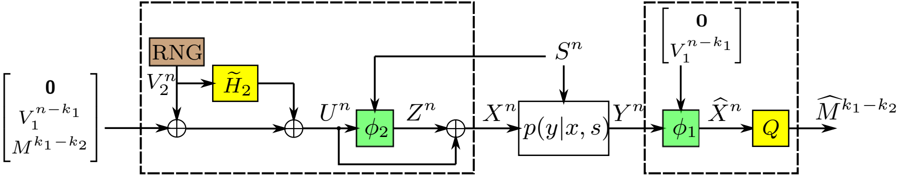

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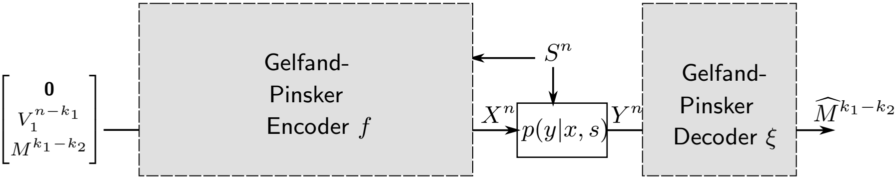

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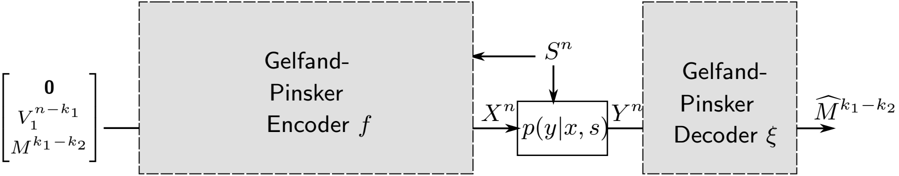

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Theorem

$R = \frac{k_1 - k_2}{n}$
$P_e \le \epsilon + \delta$

Rate Achievability

- $\exists$ a sequence of Slepian–Wolf codes for $p(x, y)$ s.t. $\epsilon \to 0$ if and only if

$$\frac{n - k_1}{n} > H(X \mid Y)$$

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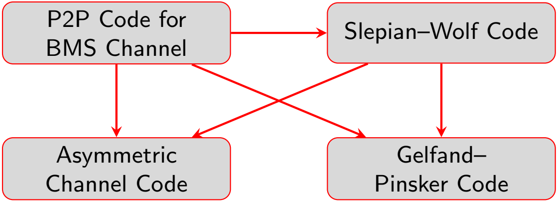
- Rate $R = \frac{k_1 - k_2}{n}$ can be made arbitrarily close to

$$H(X \mid S) - H(X \mid Y) = I(X; Y) - I(X; S)$$

Outline

- This talk:



Properties

Nested Linearity
Error Probability
Decoding Distance

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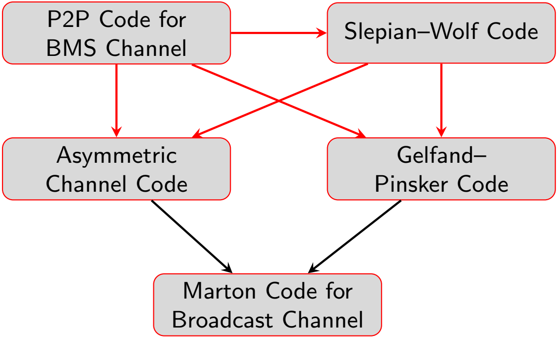

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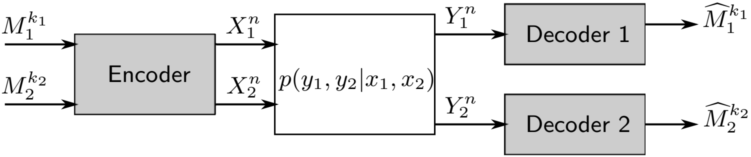
Marton Coding for Broadcast Channels

- Goal: Code for 2-user broadcast channel $p(y_1, y_2 \,|\, x_1, x_2)$ with two transmit antennas



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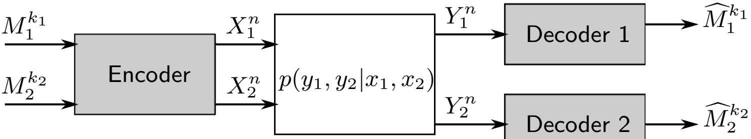

$R_1 = k_1/n$
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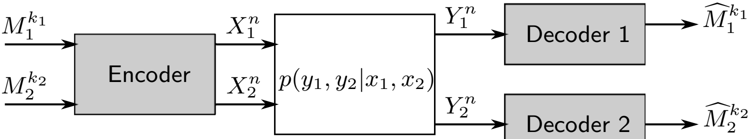

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- [Marton 1979]: $\exists$ encoder and decoders with vanishing P_e for rates (R_1, R_2) if

$$\begin{aligned} R_1 &< I(X_1; Y_1) \\ R_2 &< I(X_2; Y_2) \\ R_1 + R_2 &< I(X_1; Y_1) + I(X_2; Y_2) - I(X_1; X_2) \end{aligned}$$

for some input distribution $p(x_1, x_2)$

Marton Coding for Broadcast Channels

- Approach: Target a channel input distribution $p(x_1, x_2)$.
- This completely defines

$$p(x_1, x_2, y_1, y_2) = p(x_1, x_2)p(y_1, y_2 \mid x_1, x_2)$$

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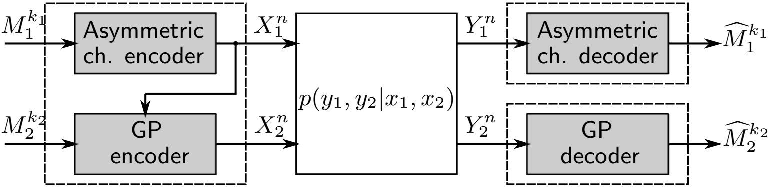

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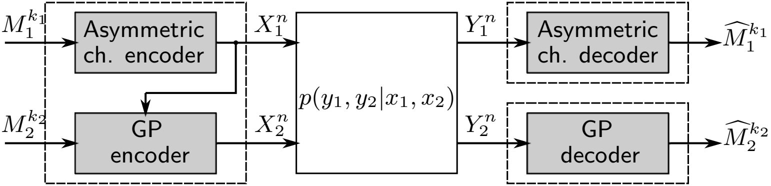

- Error probability $P_e \leq \epsilon_1 + \epsilon_2$
- Can achieve a corner point in Marton's rate region

Outline

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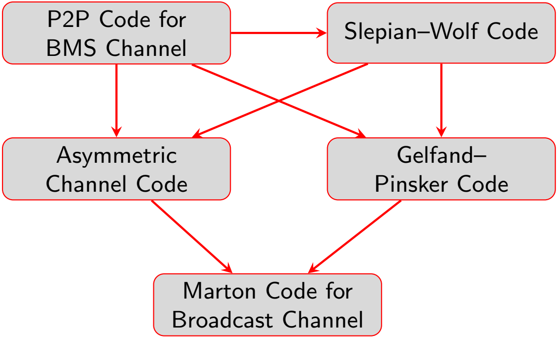

Properties

Nested Linearity
Error Probability
Decoding Distance

- Marton code can be implemented using **four** P2P codes for BMS channels

Simulation Results: Marton Coding

- Two-user broadcast channel

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = H_{\text{ch}} W \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix},$$

  where:
  - $(X_1, X_2) \in \{\pm 1\}^2$
  - $H_{\text{ch}} = \begin{bmatrix} 1 & g \\ g & 1 \end{bmatrix}$ is the channel gain matrix
  - W is a 2×2 precoding matrix used by the transmitter (if needed)
  - $Z_1, Z_2 \sim \mathcal{N}(0, 1)$ are independent
  - Encoder is subject to a sum-power constraint P such that

$$\mathsf{E}[\|W X_1^2\|^2] \leq P$$

- We use $g = 0.9$.
- Marton Coding: target a channel input distribution $p(x_1, x_2)$

Coding Strategies Over Broadcast Channel

- Maximum achievable sum-rate:

$$R_{\mathrm{sum,max}}\big(p(x_1, x_2), W\big) \triangleq I(X_1; Y_1) + I(X_2; Y_2) - I(X_1; X_2)$$

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     $\Rightarrow$ Set $p(x_1,x_2) = 1/4$ for each (x_1,x_2), and target W that maximizes $R_{\mathrm{sum,max}}$
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     $\Rightarrow$ Set $p(x_1, x_2) = 1/4$ for each (x_1, x_2), and $W = (H_{\text{ch}}^T H_{\text{ch}} + \frac{2}{P}\mathbf{I})^{-1} H_{\text{ch}}^T$
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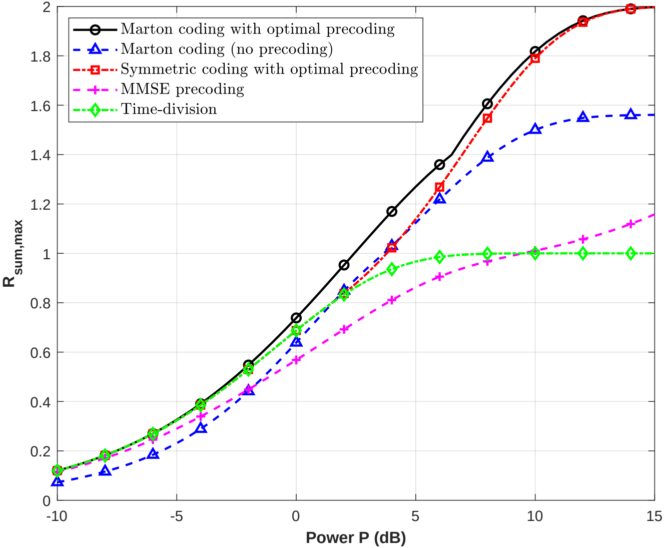
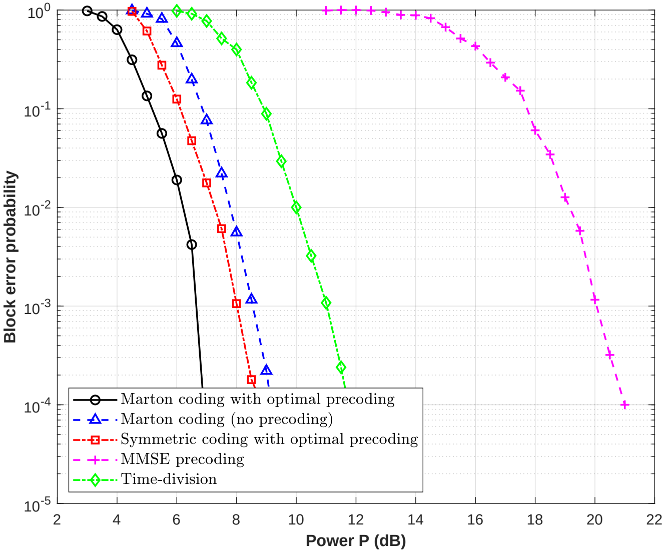
  5. Time division
     $\Rightarrow$ communicate to only one user in the channel

Maximum Achievable Sum-Rates

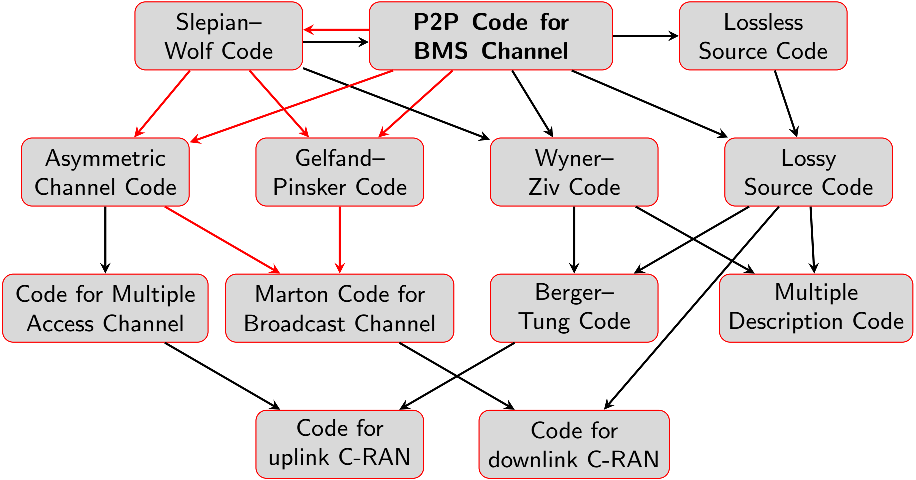


Simulations, $n = 1024$, $R_{\text{sum}} = 1$

- Use polar codes with SC decoding (rates chosen "close" to theoretical limits)



Code Constructions



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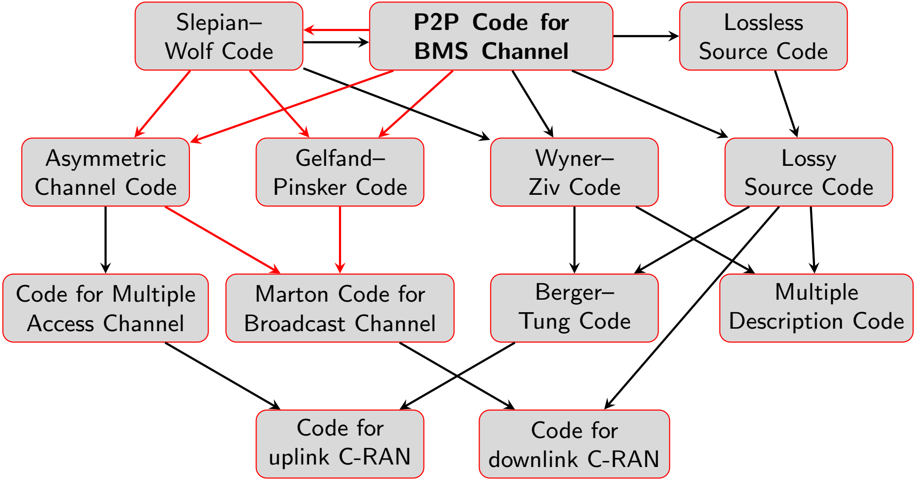
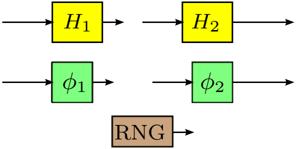


- All coding schemes can be constructed starting from P2P codes for BMS channels.
- All constructions are rate-optimal if the constituent Lego bricks are rate-optimal.*

Concluding Remarks

- Lego-brick code design:



Concluding Remarks

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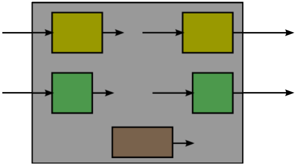
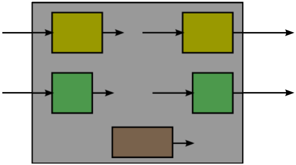
Code for a network



Concluding Remarks

- Lego-brick code design:

Code for a network



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Error Probability
Decoding Distance

Concluding Remarks

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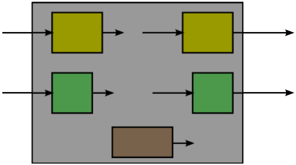

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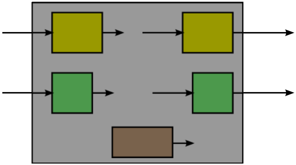

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- ArXiv paper: https://arxiv.org/abs/2211.07208

- Simulation code: https://github.com/nadimgh/lego-brick

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- **Cousins:** Manal, Ali, Leila
- **My heart:** Baba, Mama, Fattouma, Hammoudi

The best