On-Grid Angle-of-Arrival Estimation in Large-Scale MIMO Systems Using Channel Codes

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Motivation

- Initial beam alignment in mmWave massive MIMO systems

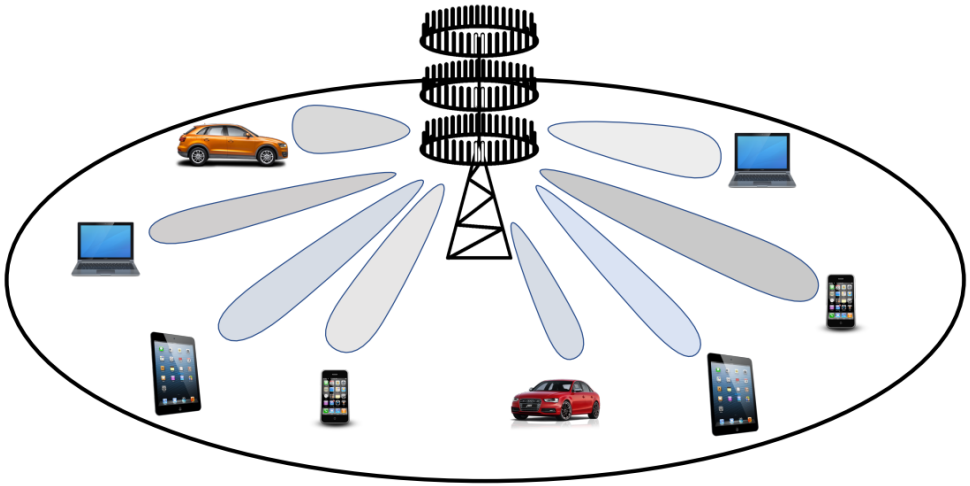


Figure: Cellular base-station with a large-scale antenna array

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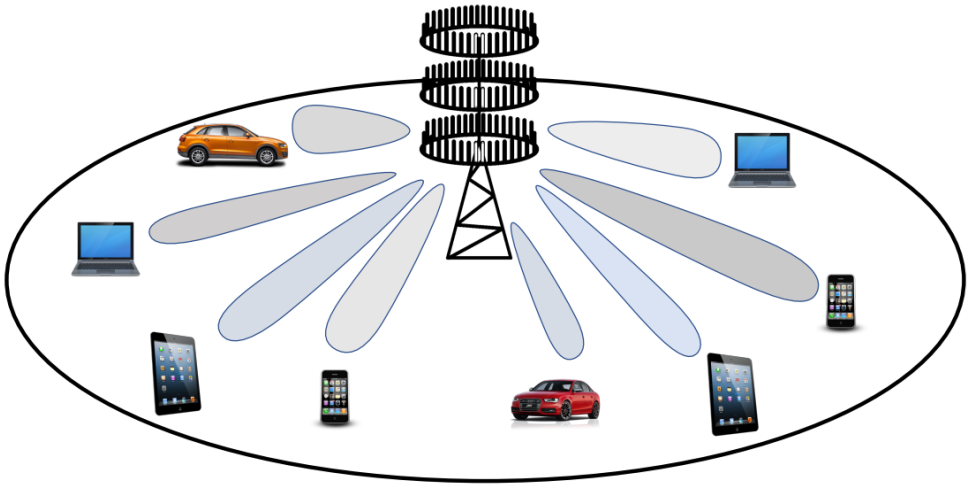


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  - Hybrid beamforming: Estimates channel using low-dimensional measurements

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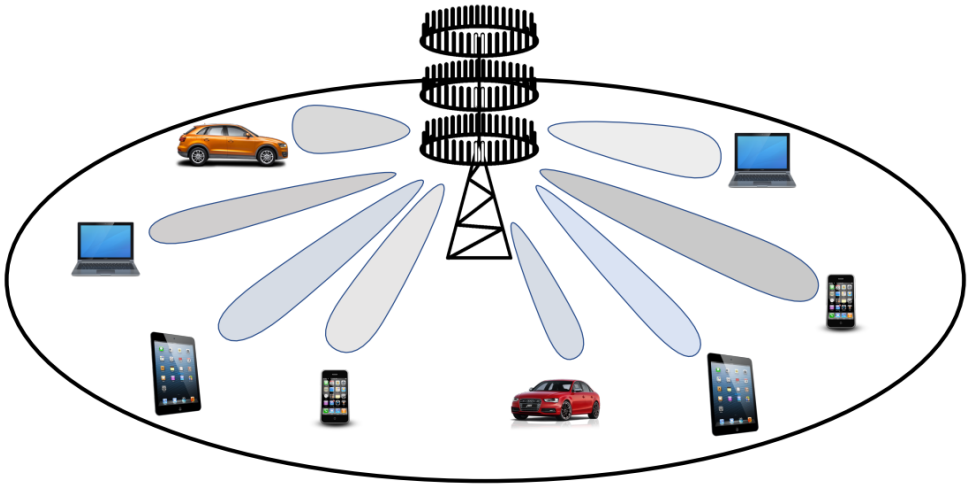


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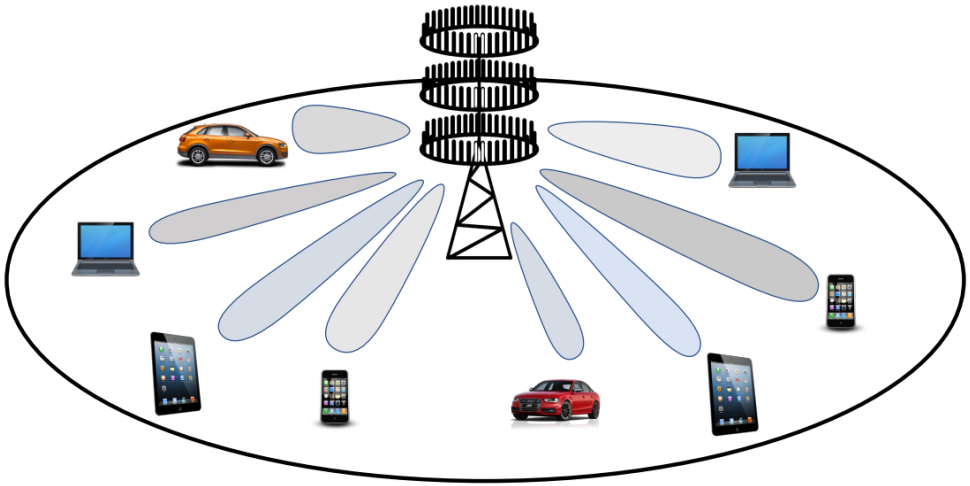


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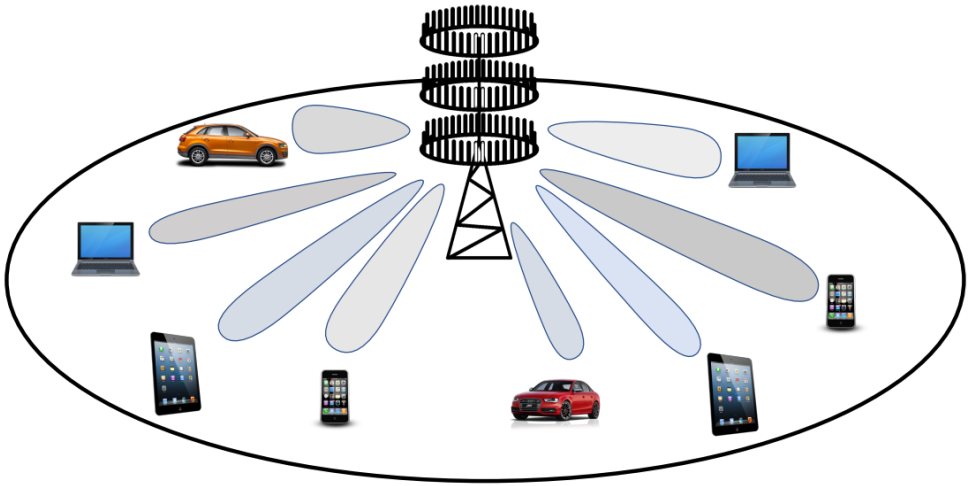
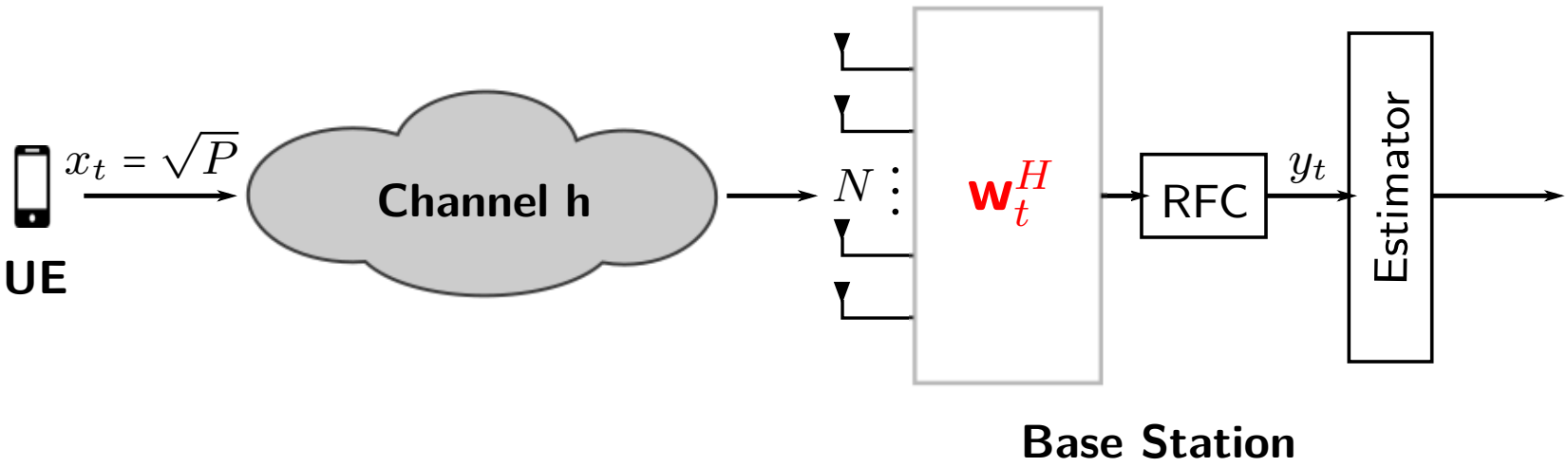


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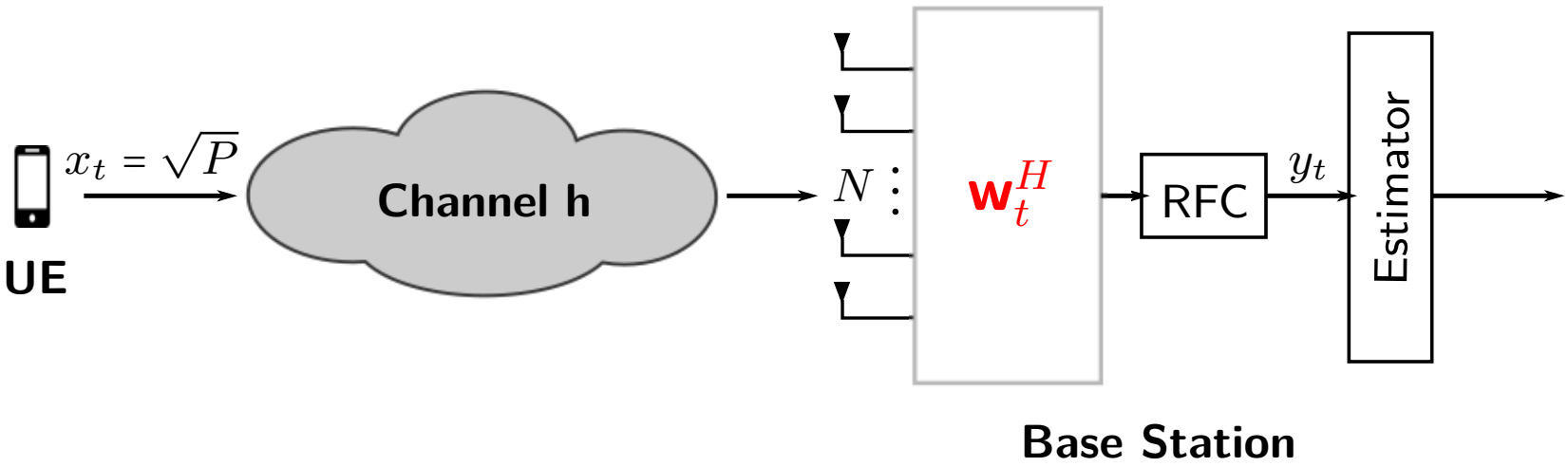
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- Question: How to estimate channel in a system with limited RF chains?
- Good news: Channel is parameterized by angles-of-arrival (AoA's) $\Rightarrow$ **AoA estimation**

Sensing Architecture with Hybrid Beamforming



- BS with N antennas and a single RF chain serves a single-antenna user
- UE sends pilots, and BS tries to estimate the channel

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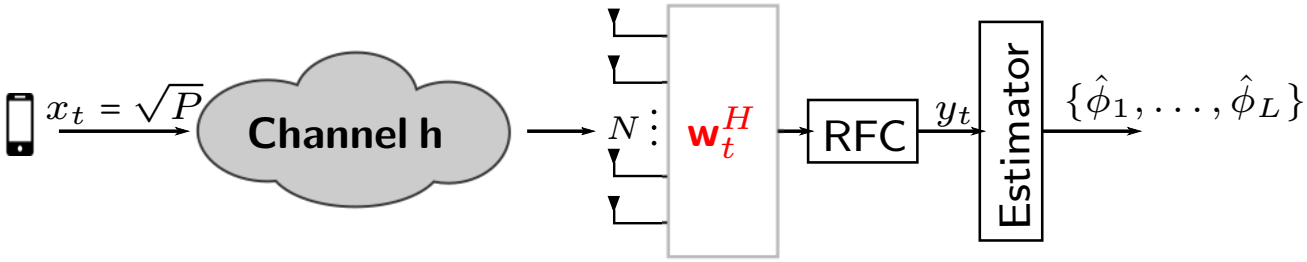


- BS with N antennas and a single RF chain serves a single-antenna user
- UE sends pilots, and BS tries to estimate the channel
- Due to single RF chain, the BS can only make a low-dimensional measurement:

$$y_t = \sqrt{P}\mathbf{w}_t^{\mathsf{H}}\mathbf{h} + \mathbf{w}_t^{\mathsf{H}}\mathbf{z}_t,$$

where $\mathbf{w}_t$ is a sensing vector at time t, and $\mathbf{z}_t \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is Gaussian noise.

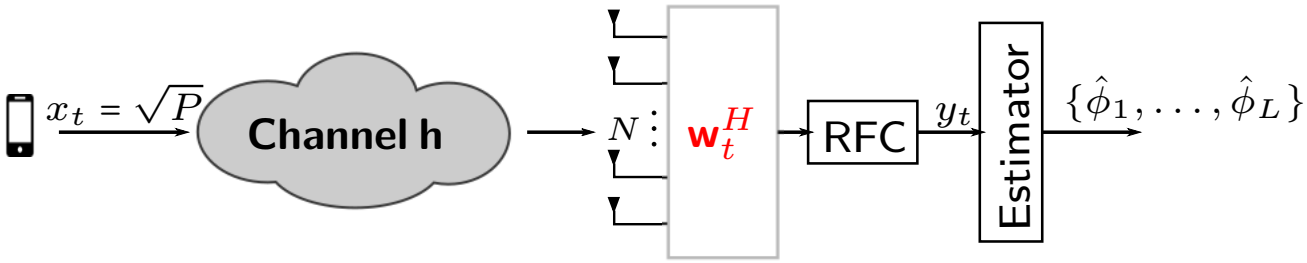
System Model



- Received symbol at time t $(1 \leq t \leq T)$:

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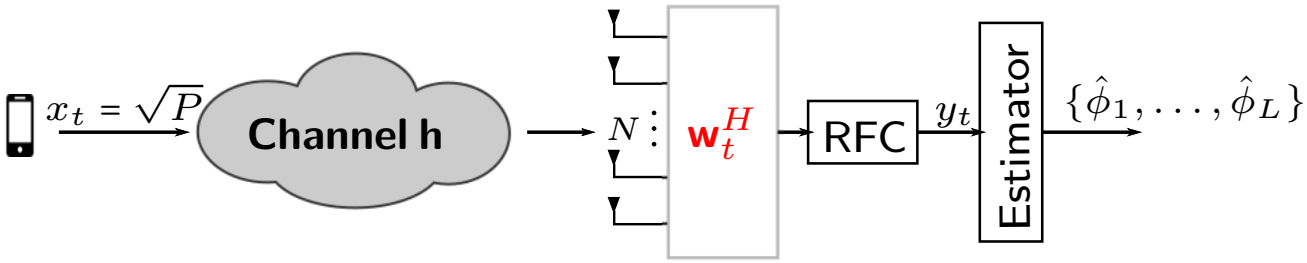
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- Channel $\mathbf{h}$ is modeled by L paths:

$$\mathbf{h} = \frac{1}{\sqrt{L}} \sum_{\ell=1}^{L} \alpha_\ell \mathbf{a}(\phi_\ell)$$

  - ϕ_ℓ is the angle-of-arrival (AoA) of the ℓth path
  - $\alpha_\ell \sim \mathcal{CN}(0,1)$ is the fading coefficient of the ℓth path
  - $\mathbf{a}(\phi_\ell) = \begin{bmatrix} 1 & e^{j\pi \sin(\phi_\ell)} & \ldots & e^{j(N-1)\pi \sin(\phi_\ell)} \end{bmatrix}^{\mathsf{T}}$ is the array response vector

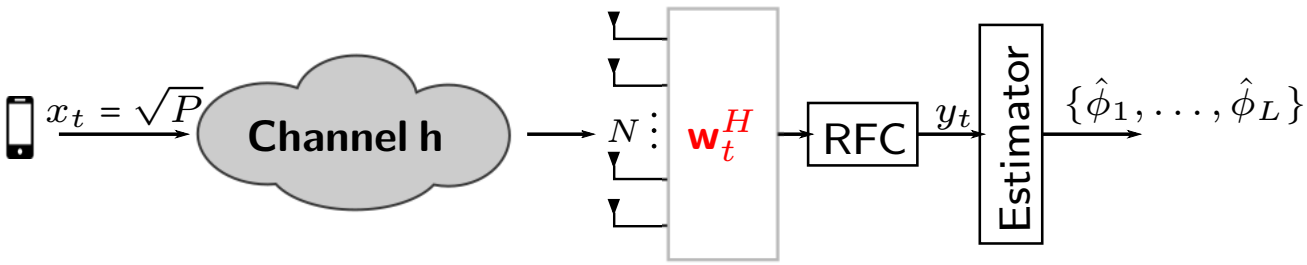
System Model



- On-grid assumption: Each ϕ_ℓ belongs to a grid Θ of M angles:

$$\Theta \triangleq \left\{ \arcsin\left(-1 + \frac{2i-1}{M}\right) : i = 1, \ldots, M \right\}$$

System Model



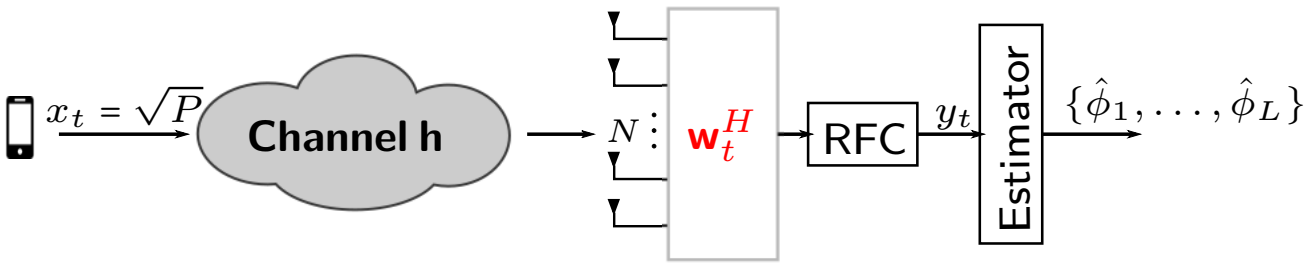
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- Accuracy of estimates is measured by MSE:

$$\text{MSE} = \frac{1}{L} \sum_{\ell=1}^{L} \mathbb{E}\left[\left(\phi_\ell - \hat{\phi}_\ell \right)^2 \right]$$

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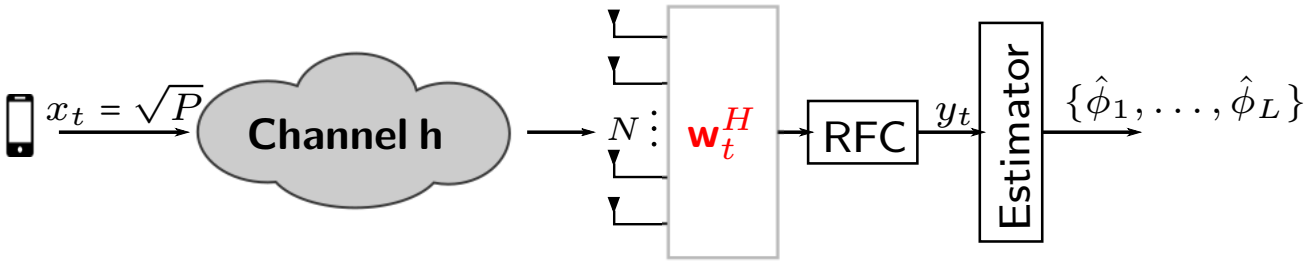
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Goal: Design $(\mathbf{w}_1, \ldots, \mathbf{w}_T)$ to get accurate estimates $\{\hat{\phi}_1, \ldots, \hat{\phi}_L\}$

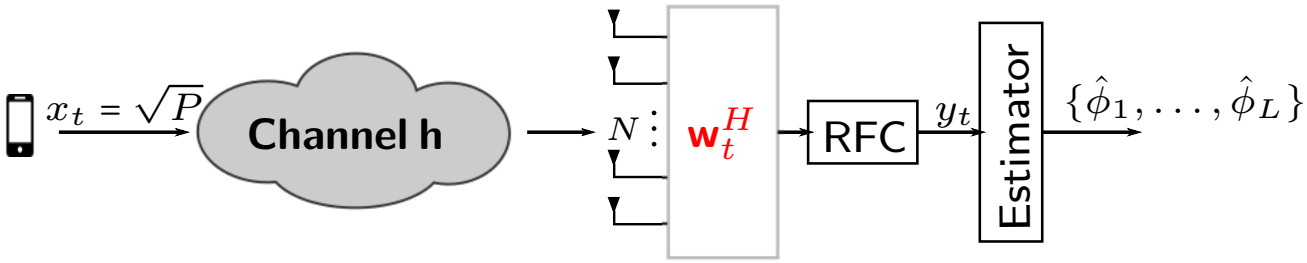
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- Example: Estimation of a single AoA:

$$y_t = \alpha\sqrt{P}\mathbf{w}_t^{\mathsf{H}}\mathbf{a}(\phi) + \mathbf{w}_t^{\mathsf{H}}\mathbf{z}_t$$

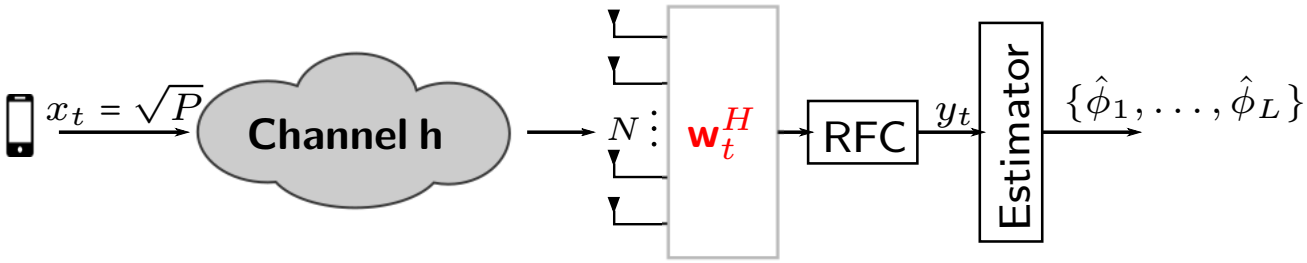
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where

$$\mathbf{D} = \begin{bmatrix} | & & | \\ \mathbf{a}(\theta_1) & \cdots & \mathbf{a}(\theta_M) \\ | & & | \end{bmatrix},$$

and $\mathbf{u}$ is 1-sparse.

Existing Beamforming Strategies

- Beam sweeping: Set $\mathbf{w}_t = \mathbf{a}(\theta_t),\ \forall\, t = 1, \ldots, M$

[1] Ahmed Alkhateeb, Omar El Ayach, Geert Leus, and Robert W. Heath. "Channel Estimation and Hybrid Precoding for Millimeter Wave Cellular Systems". In: *IEEE J. Sel. Topics Signal Process.* 8.5 (2014), pp. 831–846.

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- This work: Design beamformers based on channel codes...

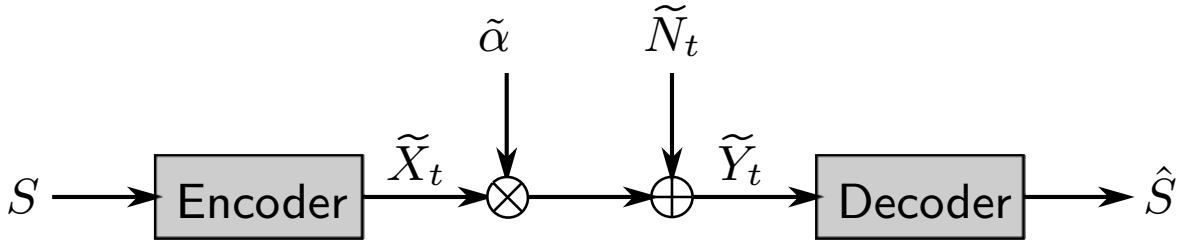
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Coding for AWGN Channels



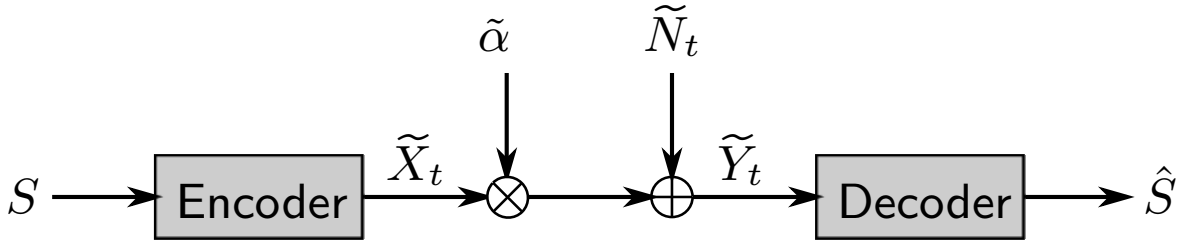
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$$\tilde{Y}_t = \tilde{\alpha}\sqrt{P}\tilde{X}_t + \tilde{N}_t,$$

where

- Message S
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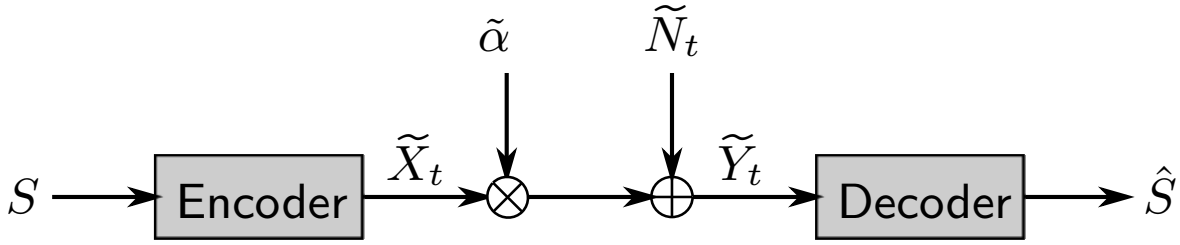
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- (R,T) code for AWGN(P) channel:
  - Message set $\mathcal{S} = [1 : 2^{RT}]$
  - Encoder $e : \mathcal{S} \to \mathbb{C}^T$ that maps S to a channel input sequence $(\tilde{X}_1, \ldots, \tilde{X}_T)$
  - Decoder $d : \mathbb{C}^T \to \mathcal{S}$ that maps $(\tilde{Y}_1, \ldots, \tilde{Y}_T)$ to a message estimate $\hat{S}$

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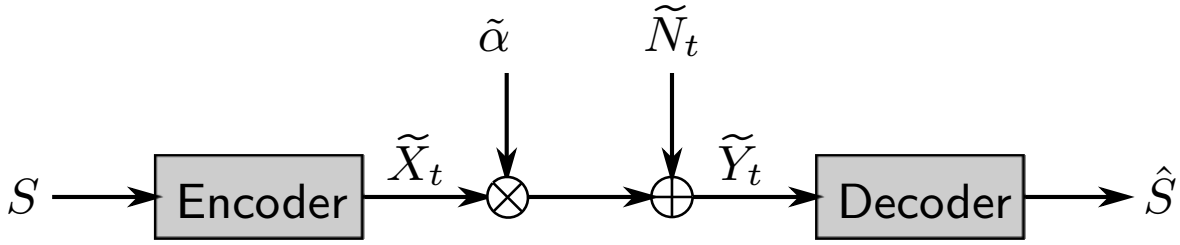
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$$P_e^{(\mathrm{AWGN})} = \mathbb{P}(S \neq \hat{S})$$

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- **Notation:** Let $\mathbf{C}$ denote the codebook matrix whose columns are the codewords

A *Code* for AoA Estimation

- Measurement model for a single AoA:

$$\begin{aligned} y_t &= \alpha\sqrt{P}\mathbf{w}_t^{\mathsf{H}}\mathbf{a}(\phi) + \mathbf{w}_t^{\mathsf{H}}\mathbf{z}_t \\ &= \alpha\sqrt{P}\mathbf{w}_t^{\mathsf{H}}\mathbf{D}\mathbf{u} + \mathbf{w}_t^{\mathsf{H}}\mathbf{z}_t \end{aligned}$$

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  - Grid Θ of angles, where $|\Theta| = M$
  - Beamformers $(\mathbf{w}_1, \ldots, \mathbf{w}_T)$ where $\mathbf{w}_t \in \mathbb{C}^N$
  - Estimator $\hat{\phi} : \mathbb{C}^T \times \mathbb{C}^{N\times T} \to \Theta$ which maps $(y_1, \ldots, y_T, \mathbf{w}_1, \ldots, \mathbf{w}_T)$ to an estimate $\hat{\phi}$

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- Probability of error metric:

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Connection Between AoA Estimation and Channel Coding

- Key connections:

AoA estimation	**Coding for AWGN channels**
AoA ϕ	Message S
Grid Θ	Message set $[1 : 2^{RT}]$
Fading coefficient α	Channel gain $\tilde{\alpha}$
Baseband source signal $\mathbf{w}_t^{\mathsf{H}}\mathbf{D}\mathbf{u}$	Channel input $\tilde{X}_t$
Probability of error $P_e^{(\mathrm{AoA})}$	Probability of error $P_e^{(\mathrm{AWGN})}$

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Probability of error $P_e^{(\mathrm{AoA})}$	Probability of error $P_e^{(\mathrm{AWGN})}$

- Key distinction:

The variance of the noise term $\mathbf{w}_t^{\mathsf{H}}\mathbf{z}_t$ in AoA estimation is $\|\mathbf{w}_t\|_2^2$.

Main Result

Theorem

Let (M, N, T) *be such that* $N \geq M$, *and let* $R = \frac{\log_2 M}{T}$. *Let* $\epsilon > 0$.

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Theorem

Let (M, N, T) be such that $N \geq M$, and let $R = \frac{\log_2 M}{T}$. Let $\epsilon > 0$.
If there exists an (R, T) code for AWGN$\left(\left\lfloor \frac{N}{M} \right\rfloor P\right)$ channel with $P_e^{(\mathrm{AWGN})} = \epsilon$, then we can construct an (M, N, T) code for AoA estimation with $P_e^{(\mathrm{AoA})} \leq \epsilon$.

A Coding-Based Beamforming Strategy

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- Underdetermined least-squares problem with solution $\mathbf{w}_t^* = \left(\mathbf{D}^{\mathsf{H}}\right)^{\dagger} \mathbf{c}_t$
- It can be shown that

$$\|\mathbf{w}_t^*\|_2^2 \leq \frac{1}{\left\lfloor \frac{N}{M} \right\rfloor}$$

Practical Considerations

1. Unknown fading coefficient α:
   - Use an estimation phase consisting of T_{est} measurements prior to AoA estimation
   - Set
   $$\mathbf{w}_t = \left(\mathbf{D}^{\mathsf{H}}\right)^{\dagger} \mathbf{1}_M$$
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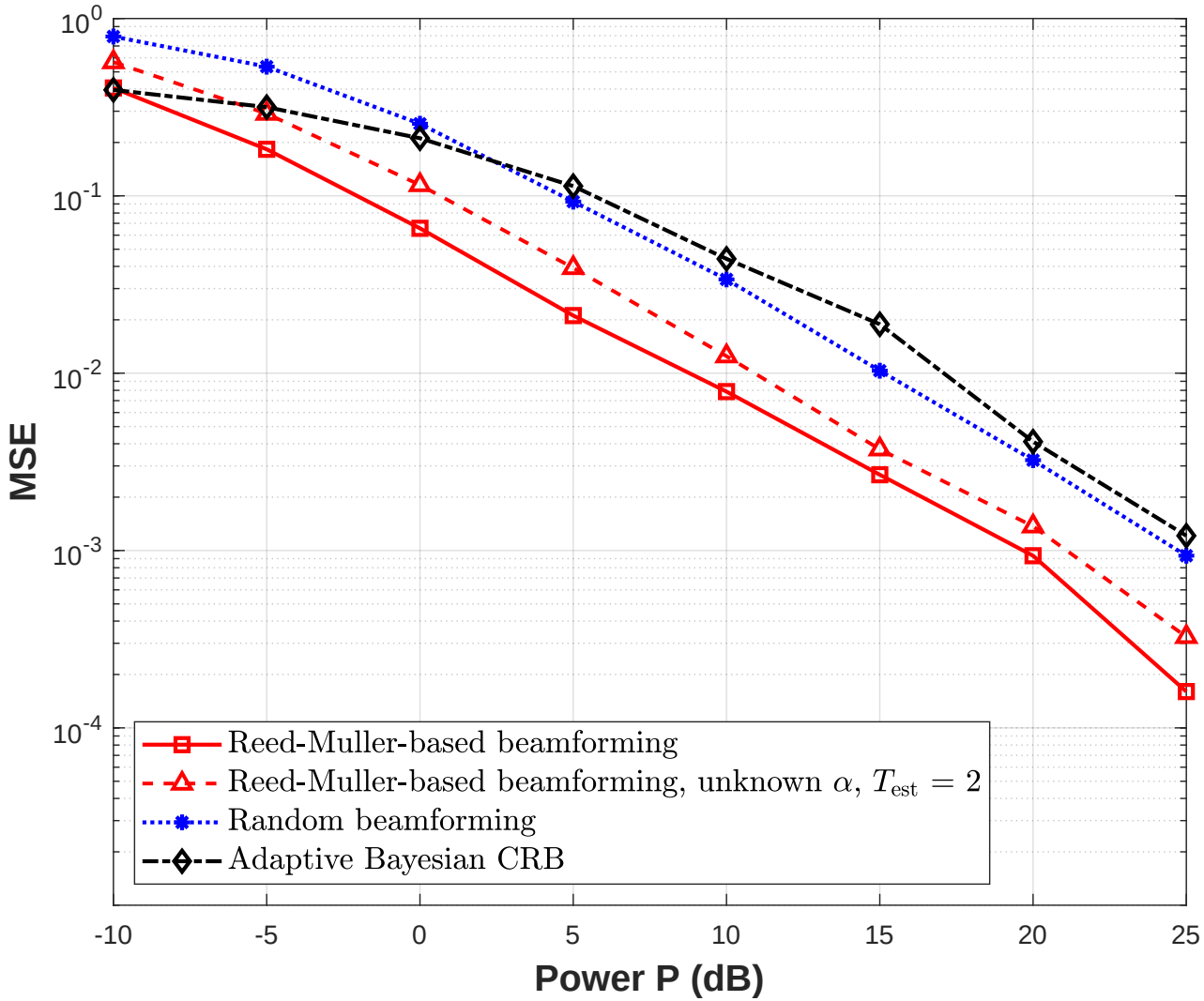
2. Estimation of multiple angles:
   - L angles $\{\phi_1, \ldots, \phi_L\}$ to be estimated
   - Connection to the L-user MAC, when users are restricted to use the same codebook
   - Use a single-user code with a successive decoder to estimate angles sequentially
   - Estimation of fading coefficients is challenging

Simulation Results - Single AoA

- $N = 1024,\ M = 256,\ T = 16,\ L = 1$
- Use a Reed-Muller code with rate $R = 1/2$

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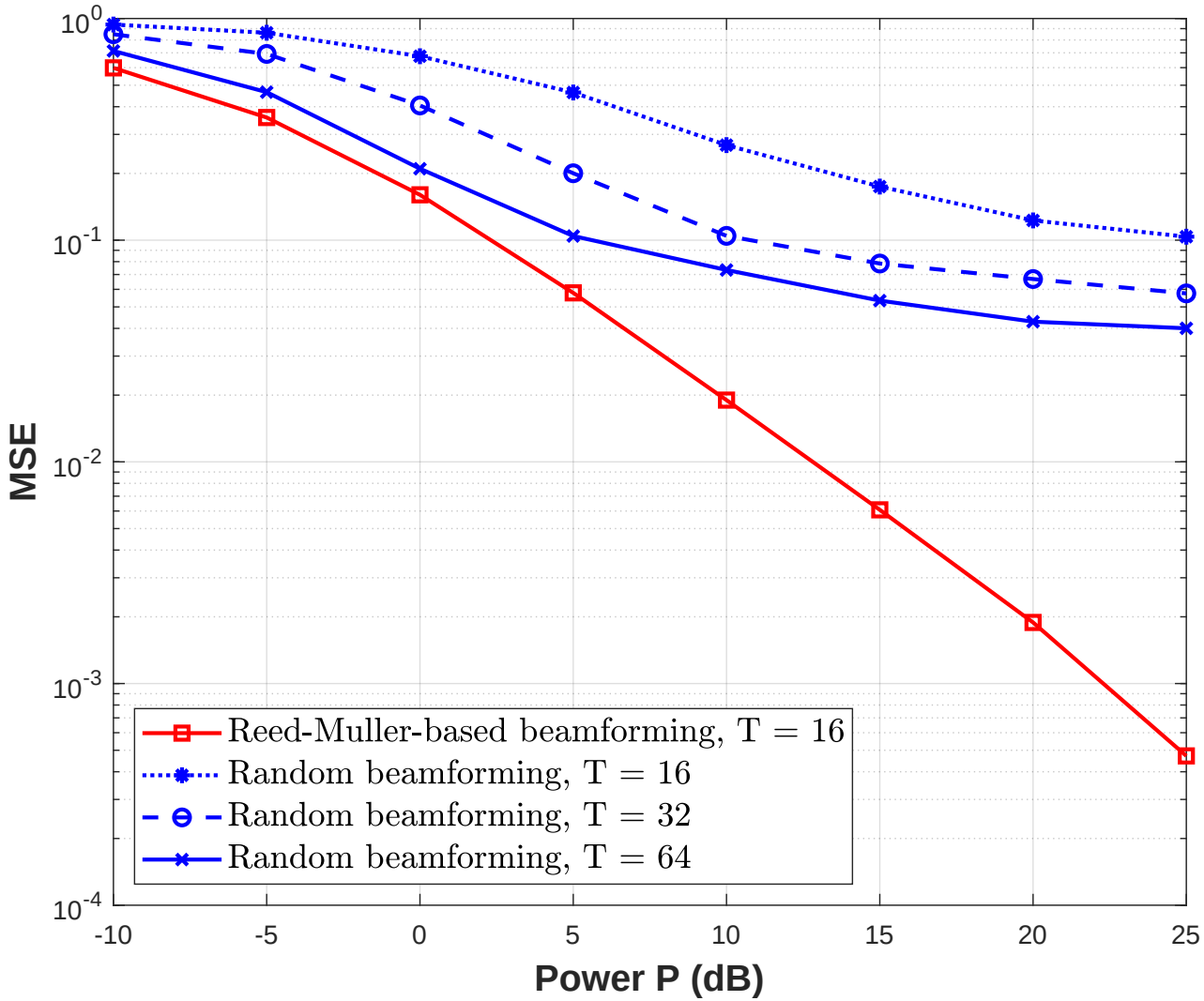


Simulation Results - Two AoA's

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Concluding Remarks

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  $\Rightarrow$ Accounts for the structure of the array response matrix
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- **Future directions:**
  1. Beamforming strategy that relaxes the condition on the number of antennas
  2. Estimation of fading coefficients in the case of multiple AoA's